**C3 January 2006**

**8.** The functions f and g are defined by $f : x \mapsto 2x + \ln 2, \; x \in \mathbb{R}$, $\quad g : x \mapsto e^{2x}, \; x \in \mathbb{R}$.

(*a*) Prove that the composite function gf is $\quad gf : x \mapsto 4e^{4x}, \qquad x \in \mathbb{R}$. **(4)**

(*b*) Sketch the curve with equation $y = gf(x)$, and show the coordinates of the point where the curve cuts the $y$-axis. **(1)**

(*c*) Write down the range of gf. **(1)**

(*d*) Find the value of $x$ for which $\frac{d}{dx}[gf(x)] = 3$, giving your answer to 3 significant figures. **(4)**

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**C3 June 2005**

**5.** (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$ **(2)**

(*b*) Show that $2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$. **(4)**

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **(4)**

(*d*) Hence, for $0 \le \theta < \pi$, solve $2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1)$,
giving your answers in radians to 3 significant figures, where appropriate. **(5)**

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**7.** The points $A$ and $B$ have position vectors $\mathbf{i} - \mathbf{j} + p\mathbf{k}$ and $7\mathbf{i} + q\mathbf{j} + 6\mathbf{k}$ respectively, where $p$ and $q$ are constants.The line $l_1$, passing through the points $A$ and $B$, has equation $\mathbf{r} = 9\mathbf{i} + 7\mathbf{j} + 7\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where $\lambda$ is a parameter.

(*a*) Find the value of $p$ and the value of $q$. **(4)**

(*b*) Find a unit vector in the direction of $\overrightarrow{AB}$. **(2)**

A second line $l_2$ has vector equation $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, where $\mu$ is a parameter.

(*c*) Find the cosine of the acute angle between $l_1$ and $l_2$. **(3)**

(*d*) Find the coordinates of the point where the two lines meet. **(5)**

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**P3 January 2006**

**4.** $$f(x) = (x^2 + 1) \ln x, \qquad x > 0.$$

(*a*) Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e. **(4)**

(*b*) Find the exact value of $\int_1^e f(x) \, dx$. **(5)**

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